Simulation and Control of a Projectile-Mounted Target Tracking Platform

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The nonlinear equations of motion for a projectile-mounted target tracking platform are developed and simulated on a digital computer. This computer simulation is used to evaluate the performance of the platform control system design. The controller is designed by linearizing the equations of motion and applying linear optimal control techniques. Application of these techniques yielded constant feedback control gains, which when multiplied by the errors of the states produced control signal inputs to the nonlinear equations of motion. The results indicate that linear control techniques can be successfully applied to this nonlinear system and that linear regulator feedback gains can be used for control of certain more general command conditions by use of feedback of the errors between command and state variables. Also indicated is that for the rolling projectile, the tracking error decreases as the projectile becomes more aligned with the target position.

Nomenclature

E_c = control voltage
I_x, I_y, I_z = moments of inertia, slug-ft^2
i = current, amps
K = solution to matrix Riccati equation
K_{emp} = back emf, volts/rad/sec
K_f = coefficient of friction, ft-lbs/rad/sec
L = inductance
N = platform normal
p,q,r = projectile roll, pitch, and yaw rates
R = target position vector
T_s = torque sensitivity, ft-lb/amp
α,β = platform pitch and yaw angles with respect to the projectile
ϵ = platform alignment error
θ = angular position of motor armature

Subscripts

0 = initial conditions
$1,2$ = of platform and gimbal ring
c,t = command or target values

I. Introduction

THE concept of missile guidance is not new, but two problems immediately arise when trying to incorporate a missile guidance system in a naval gun projectile: the first is the physical size limitation (typically the guidance system must be small enough to fit into a five-in.-diam shell); the second is that the gyros usually used in the guidance systems are very delicate and cannot withstand the extreme accelerations experienced in projectiles fired from a gun (approximately 8000 *g's* at firing).[1] It is for these reasons that projectile guidance has not been widely used.

The development of the VYRO unit by the General Electric Corporation has helped overcome the problems of extreme accelerations and of physical size.[2] The VYRO unit consists of a small metal beam supported at its free-free nodal points, and four piezoelectric transducers which sense beam motion. The VYRO is capable, without any moving parts, of sensing angular rates.

A projectile guidance system using three of these VYRO units to sense the projectile inertial rates operates as follows. The projectile is fired in the best trajectory, predicted at the time of launch, toward the target. A laser beam is then trained on the target from the launching vehicle or some other location. Located in the nose area of the projectile is a gimbaled target detector (Fig. 1) which can detect the direction of the laser reflected from the target, if it is within a 45-deg cone. When the projectile passes the apogee and starts downward, the detector picks up sight of the reflected laser beam from the target and is then driven by motors, controlled by an onboard hardwired controller, to align itself and maintain alignment with the target. Once the detector platform is aligned, its position with respect to the projectile is used as an input to a guidance scheme (typically based on target position and relative rates), which directs the projectile toward the target.

The problem is to develop the equations of motion of the target detector platform and to design a simple controller which will acquire and maintain the detector platform alignment with the target. In the development discussed below there will be no treatment of the projectile dynamics or the control of the projectile itself, the projectile motion being considered as known inputs to the tracking platform. This paper will demonstrate the ability of a simple control system based on linear analysis to achieve the above goals.

In what follows, the equations of motion are developed by energy methods. These equations in turn are linearized and are used in conjunction with linear optimal control theory to determine the feedback gains necessary to drive the target platform to alignment and maintain this alignment. A digital simulation of the complete nonlinear system with the above feedback gains is then evaluated.

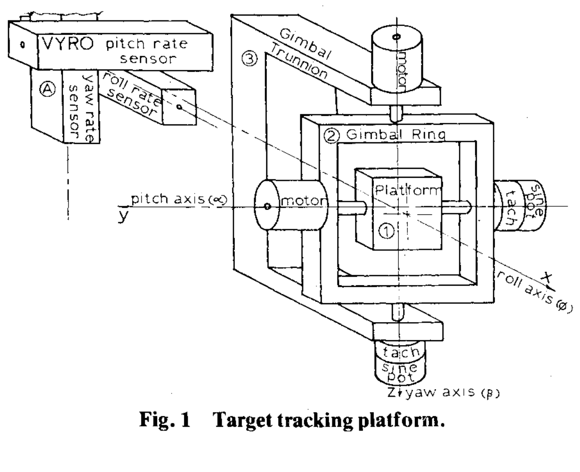

Fig. 1 Target tracking platform.

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II. Development of the Equations of Motion

In the development of the equations of motion of the target tracking platform it is assumed that there are no offset masses.

As a consequence of this assumption there is no relative translation of the platform assembly center of mass with respect to the platform axes. Also to be noted is that the projectile angular rates from the VYRO rate sensors are treated as known inputs.

The equations of motion of the target tracking platform were derived by energy methods using Lagrange's equations.[3]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial_{q_i}} = Q_i \tag{1}$$

where q_i are the generalized coordinates, Q_i are the generalized forces, and T is the kinetic energy of the system. The total kinetic energy of the system is given by (see Fig. 1)

$$\begin{aligned} T = \tfrac{1}{2}[& I_{x_1}(p\cos\alpha\cos\beta + q\cos\alpha\sin\beta - r\sin\alpha - \dot{\beta}\sin\alpha)^2 \\ & + I_{y_1}(q\cos\beta - p\sin\beta + \dot{\alpha})^2 \\ & + I_{z_1}(p\sin\alpha\cos\beta + q\sin\alpha\sin\beta + r\cos\alpha + \dot{\beta}\cos\alpha)^2 \\ & + I_{x_2}(p\cos\beta + q\sin\beta)^2 + I_{y_2}(q\cos\beta - p\sin\beta)^2 \\ & + I_{z_2}(r + \dot{\beta})^2] \end{aligned} \tag{2}$$

The generalized forces (Q_i) on the right-hand side of the equations include mechanical friction of the gimbals, internal friction of the motors, and the torque of the motors. The frictional forces are approximated as rate-dependent forces, the mechanical and motor friction being lumped together as a frictional coefficient K_f yielding a motor and mechanical friction term of $-K_{f_1}\dot{\alpha}$ for the platform and $-K_{f_2}\dot{\beta}$ for the gimbal ring. The motor torques are approximated from the dc motor armature formulation.[4]

$$Ri + L\frac{di}{dt} = E_c - K_{\text{emf}}\frac{d\theta}{dt}$$

If the electrical time constant (L/R) is small, the torque of the motors can be approximated by the torque sensitivity of the motor (T_s) multiplied by the resulting current

$$\text{torque} = T_s\frac{E_c - K_{\text{emf}}\dot{\theta}}{R} \tag{3}$$

With α and β as the generalized coordinates, Eqs. (1-3) yield the desired equations of motion.

For α

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) - \frac{\partial T}{\partial \alpha} = T_s\frac{E_{c_1} - K_{\text{emf}\dot{\alpha}}}{R} - K_{f_1}\dot{\alpha} \tag{4}$$

For β

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\beta}}\right) - \frac{\partial T}{\partial \beta} = T_s\frac{E_{c_2} - K_{\text{emf}\dot{\beta}}}{R} - K_{f_2}\dot{\beta} \tag{5}$$

III. Linearization of the Equations of Motion

The nonlinear equations of motion can be linearized about some reference equilibrium condition so that linear optimal control techniques can be applied. The reference equilibrium condition chosen is that of constant roll rate, the normal to the platform aligned with the x axis of the projectile, and all other parameters at equilibrium equal to zero, that is

$$\alpha_e = \dot{\alpha}_e = 0 \qquad \beta_e = \dot{\beta}_e = 0 \qquad q_e = \dot{q}_e = r_e = \dot{r}_e = \dot{p}_e = 0$$

and p_e = constant.

This nominal equilibrium condition was chosen because the motion of the platform relative to the projectile as it tracks a target is oscillatory in nature as the projectile rotates, therefore the oscillating platform has an average position which is normal to the x axis of the projectile.

Introducing small disturbances of

$$\alpha, \dot{\alpha}, \beta, \dot{\beta}, q, \dot{q}, r, \dot{r}, \dot{p}, p_e + \Delta p$$

into the nonlinear equations of motion, and making the usual small angle assumptions, the linearized equations of motion become

$$\ddot{\alpha} = C_1\dot{\alpha} + C_2\alpha + C_3\dot{\beta} + C_4 r + C_5\dot{q} + b_1 E_{c_1} \tag{6}$$

and

$$\ddot{\beta} = C_6\dot{\beta} + C_7\beta + C_8\dot{\alpha} + C_9 q + C_{10}\dot{r} + b_2 E_{c_2} \tag{7}$$

or in the form $\dot{x} = Ax + Bu + D\omega$ of

$$\begin{Bmatrix} \frac{d\alpha}{dt} \\ \frac{d\dot{\alpha}}{dt} \\ \frac{d\beta}{dt} \\ \frac{d\dot{\beta}}{dt} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ C_2 & C_1 & 0 & C_3 \\ 0 & 0 & 0 & 1 \\ 0 & C_8 & C_7 & C_6 \end{bmatrix} \begin{Bmatrix} \alpha \\ \dot{\alpha} \\ \beta \\ \dot{\beta} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ b_1 & 0 \\ 0 & 0 \\ 0 & b_2 \end{bmatrix} \begin{Bmatrix} E_{c_1} \\ E_{c_2} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_5 & C_4 & 0 \\ 0 & 0 & 0 & 0 \\ C_9 & 0 & 0 & C_{10} \end{bmatrix} \begin{Bmatrix} q \\ \dot{q} \\ r \\ \dot{r} \end{Bmatrix} \tag{8}$$

where the constants C_i are given in Table 1.

Selecting typical parameters,[5,6] the system matrix becomes

$$\begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ -31.55 & -1.698 & 0.0 & 1.256 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & -0.3925 & -30.22 & -0.531 \end{bmatrix}$$

and the control matrix is

$$\begin{bmatrix} 0.0 & 0.0 \\ 77.16 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 24.11 \end{bmatrix}$$

IV. Design of Controller

The objective of the controller design is to drive the absolute tracking error, ϵ, to zero. This angle ϵ is the angle between the target position vector $\boldsymbol{R}$ and the normal to the platform N (Fig. 2). The angle ϵ is found by

$$\cos\epsilon = \frac{\boldsymbol{R}\cdot\boldsymbol{N}}{|\boldsymbol{R}|\ |\boldsymbol{N}|}$$

or

$$\begin{aligned} \epsilon = \cos^{-1}(& \cos\alpha_c\cos\beta_c\cos\alpha\cos\beta \\ & + \cos\alpha_c\sin\beta_c\cos\alpha\sin\beta + \sin\alpha_c\sin\alpha) \end{aligned}$$

Table 1 Equation constants

$$C_1 = -\frac{(K_{f_1} + T_s K_{\mathrm{emf}}/R)}{I_{y_1}} \qquad C_2 = -\frac{(I_{x_1} - I_{z_1})p_e^2}{I_{y_1}}$$

$$C_3 = -\frac{(I_{x_1} - I_{z_1} - I_{y_1})p_e}{I_{y_1}} \qquad C_4 = -\frac{(I_{x_1} - I_{z_1})p_e}{I_{y_1}}$$

$$C_5 = -1 \qquad C_6 = +\frac{(K_{f_2} + T_s K_{\mathrm{emf}}/R)}{I_{z_2} + I_{z_1}}$$

$$C_7 = -\frac{[(I_{x_2} + I_{x_1}) - (I_{y_2} + I_{y_1})]}{I_{z_2} + I_{z_1}} p_e^2 \qquad C_8 = -\frac{(I_{y_1} + I_{z_1} - I_{x_1})}{I_{z_2} + I_{z_1}} p_e$$

$$C_9 = -\frac{[(I_{y_2} + I_{y_1}) - (I_{x_2} + I_{x_1})]}{I_{z_2} + I_{z_1}} p_e \qquad C_{10} = -1$$

$$b_1 = T_s \frac{E_{c_1}}{I_{y_1} R} \qquad b_2 = T_s \frac{E_{c_2}}{(I_{z_2} + I_{z_1})R}$$

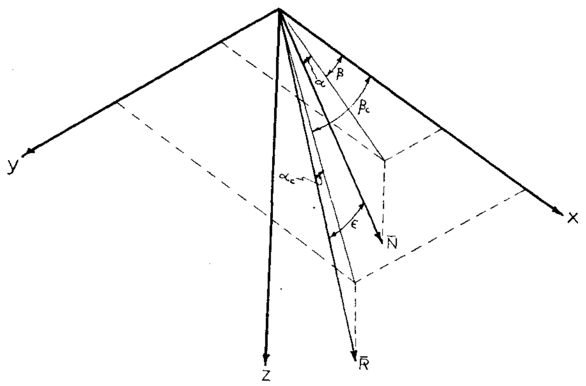

Fig. 2 Angular relations for platform and target.

For the case where the target is aligned with the missile x axis, the values of α_c and β_c are zero and the problem is cast in the form of a linear optimal regulator problem. Although the linear regulator gains obtained in this manner are not optimal solutions to the general nonlinear problem or the problem where α_c and β_c are arbitrary functions of time (tracking problem), in the interest of simplicity of implementation, these gains make suitable candidates for the desired controller. The controller has the form

$$\begin{Bmatrix} E_{c_1} \\ \\ E_{c_2} \end{Bmatrix} = [G] \begin{Bmatrix} \alpha_c - \alpha \\ \dot{\alpha}_c - \dot{\alpha} \\ \beta_c - \beta \\ \dot{\beta}_c - \dot{\beta} \end{Bmatrix}$$

where G is the matrix of feedback gains obtained from the linear optimal regulator problem presented below, and the errors $\dot{\alpha}_c - \dot{\alpha}$, $\alpha_c - \alpha$, $\beta_c - \beta$, and $\dot{\beta}_c - \dot{\beta}$ are actual output signals from the detector.

For the case where α_c and β_c are zero and the terminal time free (not specified) the optimal control law reduces to linear, asymptotic steady-state feedback of the form

$$u(t) = -R^{-1}B^T K x(t) \tag{9}$$

where the process to be controlled is described by the state equation

$$\dot{x}(t) = Ax(t) + Bu(t)$$

which is Eq. (8) with $\omega = 0$,‡ and the performance measure to be minimized is

$$J = \int_{t_0}^{t_f} (x^T Q x + u^T R u)\,\mathrm{d}t$$

The choice of the weighting matrices Q and R was made by initially selecting them to be diagonal with the element associated with the ith variable equal to the square of the reciprocal of the maximum desired error of that variable. These values were reduced until the matrix Ricatti solution converged. From here the values were varied and the changes in the closed loop eigenvalues observed. For the case where the weight on the rates was nonzero, the response exhibited nonoscillatory behavior. For the zero weights a satisfactory oscillatory response was obtained. The weights on the position were then adjusted until a satisfactory time response was obtained.

For this problem the weighting matrices were selected as

$$R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 400 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

The optimal control policy can be determined from Eq. (9), and is given by

$$G = R^{-1}B^T K = \begin{bmatrix} 62.8 & 1.25 & -0.57 & 0.0 \\ 0.57 & 0.0 & 62.0 & 2.25 \end{bmatrix} \tag{11}$$

These gains result in an augmented system matrix which yields a time to half-amplitude of 0.025 sec and a damping ratio of 0.7. The weights on the controls, Eq. (10), used to determine the controller gains, Eq. (11), assure that excessive control will not be used, but these weights do not bound the controls. In the simulation control bounds of -10 volts $\leq E_c \leq 10$ volts are used.

‡It should be noted that in this problem projectile pitch and yaw rates simply provide relative target motion. From here on we selected q and r to be zero and considered target motion only.

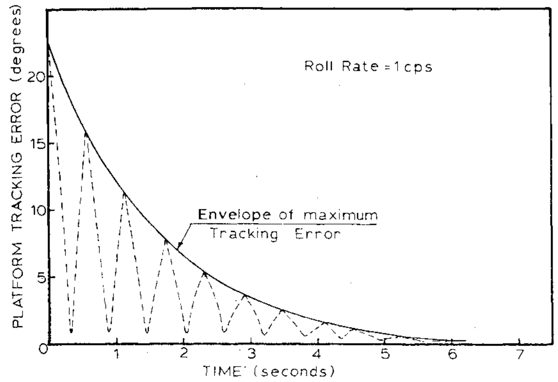

Fig. 3 Uncontrolled platform motion, target straight ahead.

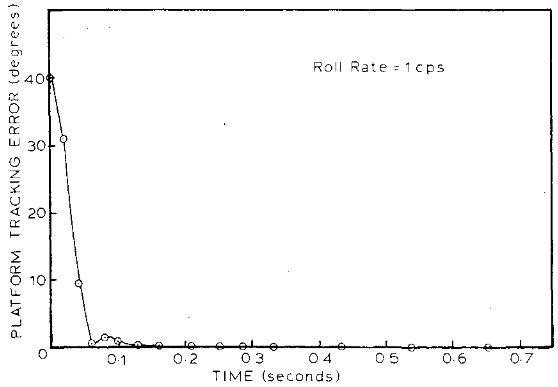

Fig. 4 Platform tracking error, regulator condition.

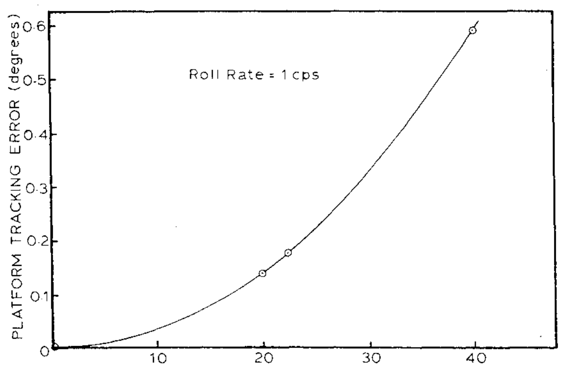

Fig. 5 Maximum platform tracking error.

V. Simulation of Target Tracking Platform

The simulation of the system was achieved by integrating the nonlinear equations of motion Eqs. (4) and (5) by a double precision version of the Hamming Predictor Corrector Method,[8] which was found to be accurate and fast. The simulation was coded in Fortran IV and executed on an IBM 370 computer. The input requirements include the desired platform angles and rates which correspond to the initial angular position and rates of the target, the actual platform initial angular position and rates, the projectile angular rates, and the optimal linear regulator gains. The output includes the actual platform angular position and rates and the absolute tracking error ϵ. Since the magnitude of ϵ indicates how closely the platform is aligned with the target, it serves as an indicator of the effectiveness of the platform controller.

In order to determine the uncontrolled system response, a simulation of the system with no control inputs was done. In the absence of control inputs the system is not affected by the target position. Therefore, the platform should oscillate around the projectile x axis with the principal axes of the platform aligned with those of the projectile due to the effect of the rolling projectile. The results are presented in Fig. 3. To evaluate the control system performance the linear regulator gains were applied to a regulator situation (target straight ahead on x axis) involving the nonlinear system simulation. The results of this activity are presented in Fig. 4. Although the linear regulator gains developed are not an optimal solution to the general nonlinear tracking problem, as previously noted, these gains were used for control in the target tracking simulation. The results of the linear controller performance for the nonlinear tracking condition are presented in Figs. 5 and 6.

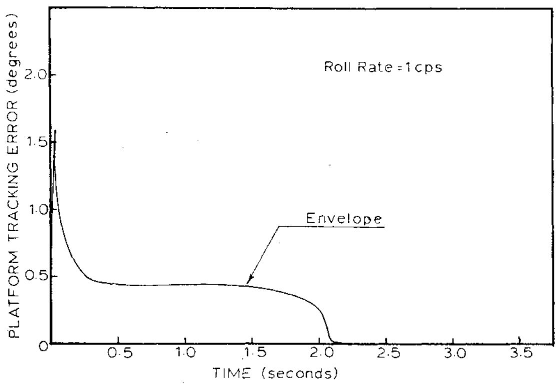

Fig. 6 Maximum tracking error with moving target.

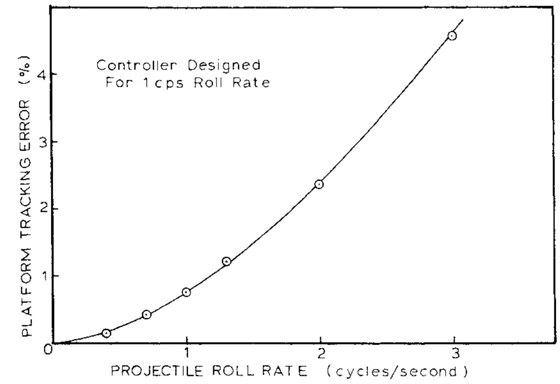

Fig. 7 Maximum tracking error for off-design roll rate.

VI. Results

To determine the uncontrolled system response to an initial offset, the platform was displaced from alignment with the projectile x axis by $\alpha_0 = 20$ deg and $\beta_0 = 10$ deg and allowed to move uncontrolled. From Fig. 3 it can be seen that the system is dynamically stable for the rolling projectile, but the time for the platform to return to alignment with the projectile x axis is quite large for this problem (a time to half-amplitude of 1.25 sec.).

The platform response, in terms of the maximum platform alignment error, using the optimal linear regulator gains applied to a regulator situation is shown in Fig. 4 (i.e., the terminal state is the origin which corresponds to a target position along the projectile x axis). In this case the platform is initially displaced by $\alpha_0 = 40$ deg. As can be seen, the platform returns to the straight ahead position in 0.347 sec and remains there. The fast response time indicates that the nonlinearity of the equations of motion due to large angles is not a significant factor when using controls that were based on linear theory which included the small-angle assumption.

It was found that for target tracking cases, where the target is at a specified angle from the projectile x axis and the platform is initially aligned with the target, the platform successfully tracks the target. For a target off-axis angle of $\alpha_t = 40$ deg, there is a small initial deviation of 2.77 deg, and the platform settles back in 0.18 sec to alignment with the target with a maximum tracking error of 0.587 deg which is 1.47% of the target off-axis angle. The variation of platform tracking error for different target off-axis angles is shown in Fig. 5.

Also investigated was the case where the target is offset relative to the projectile x axis and the platform is initially aligned with the projectile x axis.

For the target at $\alpha_t = 40$ deg the platform aligns with the target within 0.22 sec and stays aligned with a slight oscillation. The maximum tracking error is 0.587 deg (as in the tracking case Fig. 5).

Figure 6 shows the ability of the platform to track a target which moves from an initial position offset from the projectile x axis to being aligned with the projectile x axis. This target motion takes place from 0 to 2 sec in a linear fashion. For $t > 2$ sec the target stays aligned with the projectile x axis. The initial target position is $\alpha_{t_0} = 20$ deg and $\beta_{t_0} = 10$ deg and moves at the rate of 11.13 deg/sec toward the projectile x axis. As can be seen the platform is aligned with the target within 0.45 degrees while the target is moving. Once the target is straight ahead the platform stays aligned with the target.

Shown in Fig. 7 is the effect of perturbations in the projectile roll rate on the ability of the platform to track the target using controller gains developed for a roll rate of 1 cps. In each case the platform is initially aligned with the target which is at $\alpha_t = 20$ deg and $\beta_t = 10$ deg. For the nominal roll rate of 1 cps the percent tracking error is the aforementioned 0.77%. For perturbations of 30% slower roll rate (0.7 cps) this error dropped to 0.427%; and for an increase in roll rate to 1.3 cps, the tracking error went up to 1.21%. As can be seen in Fig. 7, an increase in roll rate to 2 cps caused the percent error to rise to 2.39% and at 3 cps the percent tracking error was 4.58%. These results indicate that the controller design should be based on the roll rate that will be experienced during the tracking phase of the projectile flight if minimum tracking error is to be achieved.

Also it was noted that the natural tendency of the platform to align with the projectile x axis at higher roll rates causes the regulator gains developed for these higher roll rates to be smaller than those developed for slower roll rates. Consequently, when these gains are applied to the problem of the platform tracking an off-axis target, the tracking error is amplified.

VII. Conclusions

From the above results, it can be concluded that for this problem the controller designed using linear optimal regulator theory for a rather restricted case provides adequate control of the nonlinear system even in the tracking situation. The linear regulator performance characteristics were good enough so that the degradation due to nonlinearities and tracking was not sufficient to warrant redesign.

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